Quantitative Prediction and Molar Description of the Environment

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Molecular explanations of behavior, based on momentary events and variables that can be measured each time an event occurs, can be contrasted with molar explanations, based on aggregates of events and variables that can be measured only over substantial periods of time. Molecular analyses cannot suffice for quantitative accounts of behavior, because the historical variables that determine behavior are inevitably molar. When molecular explanations are attempted, they always depend on hypothetical constructs that stand as surrogates for molar environmental variables. These constructs allow no quantitative predictions when they are vague, and when they are made precise, they become superfluous, because they can be replaced with molar measures. In contrast to molecular accounts of phenomena like higher responding on ratio schedules than interval schedules and free-operant avoidance, molar accounts tend to be simple and straightforward. Molar theory incorporates the notion that behavior produces consequences that in turn affect the behavior, the notion that behavior and environment together constitute a feedback system. A feedback function specifies the dependence of consequences on behavior, thereby describing properties of the environment. Feedback functions can be derived for simple schedules, complex schedules, and natural resources. A complete theory of behavior requires describing the environment's feedback functions and the organism's functional relations. Molar thinking, both in the laboratory and in the field, can allow quantitative prediction, the mark of a mature science.

Key words: molar description, feedback function, behavior-environment system, operant behavior, hypothetical constructs

Like any experimental science, the science of behavior is judged by its ability to allow prediction and control. Sometimes qualitative prediction and control suffice: in clinical settings, classrooms, and situations where one requires only that some behavior decrease and other behavior increase. Some applications, however, require quantitative prediction. In the field of organizational behavior management (OBM), for example, particularly in deciding whether some technique of behavioral change is cost-effective, one wishes to know not only whether behavioral output will increase but *how much* it will increase. Similar questions arise in behavioral ecology, which has applications in wildlife management. There, one wishes to predict how much of a resource an organism is likely to consume, and how much might be left at the end of its predation (see, e.g., Taylor, 1984; Stephens & Krebs, 1986).

Apart from these practical concerns, quantitative prediction is generally more satisfying than qualitative, because quantitative prediction is widely considered the mark of a mature science. If we can tell only what sorts of changes should occur, we are at a more primitive scientific stage than if we can also tell how much change should occur.

The basic requirement for quantitative prediction is that one be able to write mathematical formulas having the general form $B = f(x)$, where B stands for response rate and x stands for an environmental independent variable. Since reinforcers and schedules of reinforcement are qualitative descriptions, they cannot stand for x. Rather, x must be a measure of some quantitative dimension of reinforcement, such as magnitude in grams or delay of reinforcement in seconds.

MOLAR *VERSUS* MOLECULAR EXPLANATION

Magnitude and delay exemplify variables that can be called *molecular,* which here will mean that they can be measured on any one occurrence of an event, such as the presentation of a reinforcer. Each time a grain magazine operates, one can measure the number of grams eaten or the delay since the last response. Molecular variables contrast with *molar* variables, which can only be measured over an aggregate of many events (e.g., presentations of a reinforcer). Rate of reinforcement, for example, cannot be measured on any one presentation of grain, but must be calculated by counting the number of presentations over some substantial period of time (Baum, 1973; Rachlin, 1976).

In what follows, I will argue that molecular variables cannot suffice for quantitative prediction. Moreover, attempts to rely on them exclusively not only fall short of quantitative prediction, but necessitate hypothetical constructs of questionable validity.

Molecular Theories

To see the inadequacy of molecular theories, let us consider explanations of two phenomena: (1) the higher response rates on ratio schedules than on interval schedules, and (2) free-operant avoidance.

The standard molecular account of the ratio-interval rate difference appeals to two factors, the strengthening effect of reinforcement on the response immediately preceding it and the differential reinforcement of inter-response times (IRTs) (e.g., Mazur, 1986). In interval schedules, the longer the time since the last response—the longer the IRT—the higher the probability of reinforcement. Thus, longer IRTs are differentially reinforced and become more frequent, lowering response rate. Acting in the opposite direction, reinforcement tends to increase response rate in both interval and ratio schedules. Since its effect is opposed by differential reinforcement of

IRTs only in an interval schedule, response rate is higher on a ratio schedule.

One shortcoming of this theory is that without further specification of how response rate depends on reinforcement, one can make no quantitative prediction. As it stands, the theory tells us that the ratio rate will be higher, but not how much higher. To make a quantitative prediction while doing without molar variables like rate of reinforcement, one would have to specify how each instance of reinforcement increments *response strength,* a hypothetical construct that would in turn determine response rate.

Explanation of the second phenomenon, free-operant avoidance, by molecular theory also requires hypothetical constructs. The molecular theory is based on the two-factor theory of signalled avoidance. Stimuli preceding an aversive event like electric shock are said to elicit "fear," and the avoidance response is reinforced by fear reduction when those stimuli are removed (Solomon & Wynne, 1954). In free-operant avoidance, where there are no exteroceptive stimuli, appeal is made to temporal regularity in the presentation of shock, which is considered to produce conditioned aversive temporal stimuli (CATS) that are supposed to elicit "fear" and allow its reduction by the avoidance response (Anger, 1963).

The molecular explanation of free-operant avoidance has the same two shortcomings as the molecular explanation of the higher response rates on ratio than interval schedules. First, the theory predicts only that avoidance responses will occur; it makes no quantitative prediction about their rate of occurrence. Second, neither "fear" nor CATS are observable, and their properties are unknown. They are hypothetical constructs, necessary only if one must avoid referring to frequency of shock, a molar variable.

Molecular theories require hypothetical constructs because the explanation of any response lies not only in events at the moment of its occurrence but in an aggregate of events over a span of time, often loosely referred to as a "history of reinforcement." A rat presses a lever when

a light is turned on because in the past when the light was on presses were reinforced with some frequency and were extinguished when the light was off. Historical variables are often molar variables; to the extent that one must explain behavior historically, one will need often to refer to molar variables.

What's Wrong with Hypothetical Constructs?

Hypothetical constructs have earned a bad name, not because they are bad in principle—atoms are, after all, hypothetical constructs—but because in practice they are usually vague and ill-defined. If they were well-defined, we would know the mathmatical relations they embody, and we could make quantitative predictions. Vague hypothetical constructs, however, allow no quantitative prediction and are probably worse than none at all.

The hypothetical constructs characteristic of molecular theories, like "strength" and "fear," serve the same sort of explanatory purpose as mental constructs like "anxiety" and "memory," to invent present causes for historically caused behavior. They arise from a common prejudice that insists on placing causes in the present. Staddon (1973) suggested that the prejudice may have originated in the course of evolution; it may be adaptive to perceive the event that immediately precedes some occurrence as the cause of the occurrence. To say that the causes of behavior are historical is to say that there is a gap, that they cannot be found immediately before the behavior, but that they can be found in the history, and that there is neither need nor use to fill the gap with fictional mediators.

To say that the causes of behavior are historical is to suggest also that they constitute an aggregate, that historical causes translate into molar environmental variables. Like mental constructs, the hypothetical constructs of molecular theories function only as *surrogates,* surrogates of molar environmental variables, such as rate of shock or rate of reinforcement. Like mental constructs, too, they are su-

perfluous, because adequate molar theories are possible without them.

Molar Theories

In contrast to molecular theories, molar theories refer to environmental variables that are physical and readily specified. Questions may arise about how to calculate rate of reinforcement—over how long a time period or by using an arithmetic or a harmonic mean of intervals between reinforcers (Killeen, 1968)—but the properties of any computation are well known, and no hypothetical constructs are needed. Moreover, whereas molecular theories exclude molar variables, molar theories in no way exclude molecular variables. Magnitude and delay may be important, as well as rate of reinforcement. And molar theories make quantitative prediction of response rate possible.

A molar theory can explain free-operant avoidance, for example, relatively simply. Once we are allowed to refer to the frequency of the aversive event, we can see that avoidance responding is maintained by the reduction it produces in that frequency (Herrnstein, 1969). Herrnstein and Hineline (1966), for example, found a direct relation between rate of avoidance responding and the amount of reduction in shock rate that responding produced, whereas they found an inverse relation between number of responses to extinction and the amount of reduction that had been possible in prior training.

THE BEHAVIOR-ENVIRONMENT FEEDBACK SYSTEM

Molecular variables by themselves fail to provide adequate accounts of behavior because behavior produces results in the environment that in turn affect behavior. That is, behavior and environment together constitute a closed feedback system. To explain behavior one must think of both behavioral output (e.g., response rate) and environmental input (e.g., rate of reinforcement) as continuous variables—that is, as flows through time.
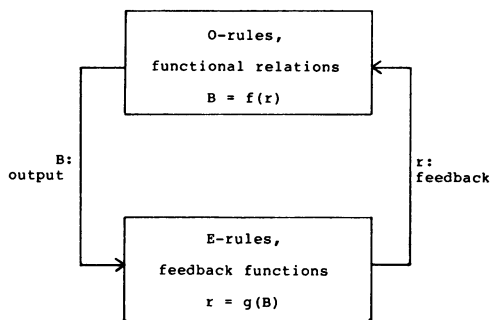
Figure 1 depicts the behavior-environ-

Figure 1. The behavior-environment feedback system.

ment feedback system in extremely simple form. The environment provides functions of the form $r = g(B)$, in which we see the dependence of some environmental variable r (e.g., rate of reinforcement) on a behavioral variable B (e.g., response rate). These functions, called "E-rules" in an earlier paper (Baum, 1973), nowadays are called feedback functions. By themselves, they tell nothing about how an organism will behave; they tell only what sort of an environment the organism is behaving in.

To form a theory of behavior, we must know what the organism brings to the situation. The organism provides functions of the form $B = f(r)$, in which we see the dependence of the behavioral variable B on the environmental variable r. These functions, called "O-rules" earlier, are the same as Skinner's functional relations (Baum, 1973).

The figure emphasizes that in operant relations feedback should be understood as axiomatic: B affects r just as much as r affects B. It illustrates also that in a feedback system the distinction between independent and dependent variables becomes arbitrary; strictly speaking, all variables depend on one another.

A reasonable research program might begin by trying to specify feedback functions and then trying to discover functional relations. Once these two goals are met, the result is a quantitative theory that allows quantitative predictions of behavior. In practice, these two attempts go on in parallel, but there is a sense in which the description of the environment—the feedback functions—might

need to come first. To learn about the organism, the experimenter systematically manipulates the environment. To specify these manipulations exactly, the feedback functions need to be worked out. Once we can be precise about the ways in which the environment varies, then we can isolate the invariances that the organism contributes to the system. For that reason, I am focusing here on feedback functions.

Simple Feedback Functions

Figure 2 shows two methods of depicting the dependence of reinforcement on behavior in interval and ratio schedules. On the left, the requirements for reinforcement are represented in cumulative-recorder coordinates. In the ratio schedule, when the cumulative record hits the horizontal line corresponding to the number of responses required by the schedule, reinforcement occurs. In the interval schedule, when the cumulative record crosses the vertical line corresponding to the time at which reinforcement becomes available, the next response produces reinforcement.

The right-hand side of Figure 2 shows feedback functions for ratio and interval schedules. Those for *ratio* schedules appear as lines through the origin, because rate of reinforcement is directly proportional to response rate in a ratio schedule; if ten responses are required for each reinforcer, then the rate of reinforcement must be one-tenth of the response rate. The proportionality varies inversely with the ratio; the larger the ratio, the flatter the line. The feedback function for an *interval* schedule approaches the programmed rate of reinforcement as an upper limit; if reinforcers are scheduled only once a minute, then they can be obtained no more frequently than once a minute. The concave curvature reveals the "diminishing returns" characteristic of interval schedules; that is, as response rate grows, increases in response rate produce smaller and smaller increases in rate of reinforcement, and beyond a certain response rate further increases produce virtually no increase in rate of reinforce-

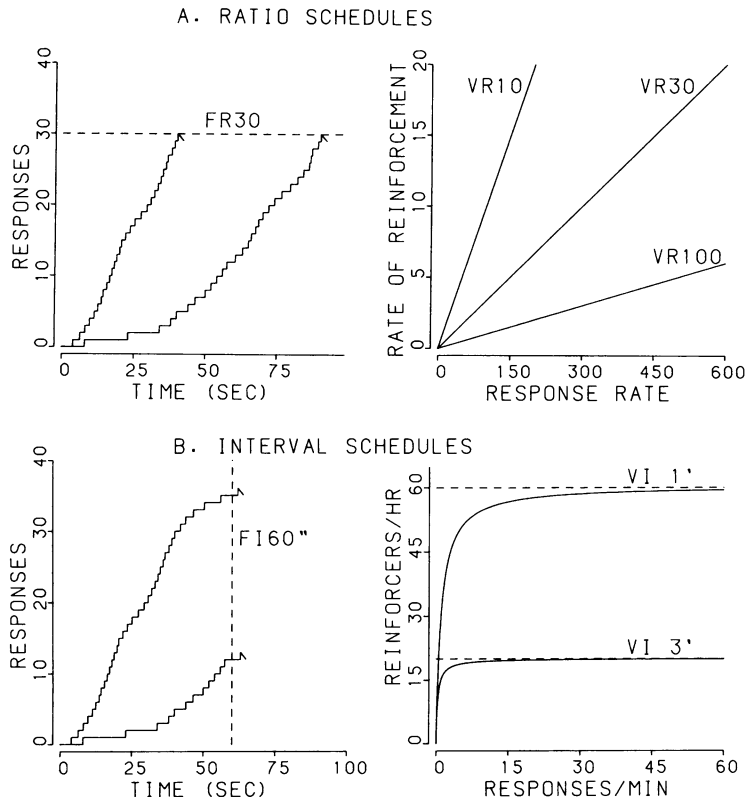## A. RATIO SCHEDULES

B. INTERVAL SCHEDULES

Figure 2. Cumulative-record representations (left) and feedback functions (right) for ratio and interval schedules.

ment. You may check your mailbox ten times a day, but if the mail is delivered once, your efforts will only produce mail once.

The difference between the feedback functions of ratio and interval schedules provides a straightforward account of the differences in performance, including the difference in response rate. The linear feedback function of the ratio schedule may be thought of as differential reinforcement of high response rate, because the higher the response rate, the higher the rate of reinforcement; hence the high rates characteristic of ratio schedules. A larger ratio gives a lower slope to the line and less differential reinforcement in the sense that each increment in rate of reinforcement is more costly with the larger ratio. If the ratio is large enough, ratio strain occurs, and responding will drop from a high level to zero; intermediate rates should not (and do not) occur (Baum, 1981). This explanation accounts

for the high work rates maintained by piecework wages (an example of a ratio schedule) and also for one of the chief objections to them: the employer is tempted to maximize profit by requiring output that just falls short of ratio strain. Molecular analysis offers no comparable account, because ratio schedules provide no differential reinforcement of IRTs; the probability that the next response will be reinforced is unaffected by the passage of time.

In the interval-schedule feedback functions shown at the lower right in Figure 2, differential reinforcement of response rate occurs only at low response rates; if you only check for mail once a month, then checking twice a month would probably produce mail twice as often. As the curve flattens out, differential reinforcement ceases at a relatively low rate; hence the lower rates characteristic of interval schedules. The higher the upper limit of the feedback function, the more slowly

the curve approaches the limit and the higher the response rate at which the curve becomes virtually flat. This shift is usually small and predicts only moderate increases in response rate with increasing rate of reinforcement when the interval schedule is shortened, in keeping with those usually observed (Baum, 1981). If, however, the scheduled upper limit were very high indeed, as in an extremely short interval schedule (e.g., variable interval 2 sec), then the rising portion of the curve would continue even into high response rates, and the result should be a transition to extreme response rates like those normally maintained by ratio schedules. In at least one experiment, I have observed such an effect (Baum, 1986).

Feedback Functions and Compound Schedules

Pure examples of ratio and interval schedules are difficult to find in the everyday world. One of the ways in which the complexity of everyday life can be approached is to combine ratio and interval requirements into compound schedules. Three types of compound schedules are conjunctive, alternative, and interlocking schedules.

Conjunctive interval-ratio schedules. Figure 3A shows, the same two ways as before, the characteristics of conjunctive interval-ratio schedules. The cumulative-record depiction of the schedule (left) shows how both the interval and the ratio requirement must be met before reinforcement can occur. If the organism responds at a high rate, it will meet the ratio requirement (horizontal line) early, but must persist until the interval requirement (vertical line) is met. If it responds at a low rate, it will fail to have met the ratio requirement when it satisfies the interval requirement, and must continue until it has met the ratio requirement. There is one response rate that satisfies both requirements simultaneously.

The feedback function (right) shows how the requirements affect the dependence of rate of reinforcement on response rate. Low response rates pay off according to the ratio schedule. High re-

sponse rates pay off according to the interval schedule. The response rate that satisfies both requirements simultaneously, B', where the two functions intersect, can, with a few additional assumptions, be shown to be the optimal performance.

This schedule might be a realistic representation of hourly wages. The interval requirement alone falls short, because employers typically expect some minimal performance even when they pay by the hour. The result is that the employee must meet a work requirement within a time schedule. Without other incentives (i.e., modification to the feedback function), the employee's optimal response rate is B'.

Alternative interval-ratio schedules. Figure 3B shows the characteristics of alternative interval-ratio schedules. The cumulative-record depiction shows how either the ratio requirement may be met by a high rate or the interval requirement may be met by a low rate. The feedback function reveals that low rates pay off according to the interval schedule, whereas high rates pay off according to the ratio schedule. There is a response rate B' that meets both requirements simultaneously, but here that rate has no special advantage. The situation offers an implicit choice: respond at high rates, above B', and take control of the rate of payoff, or respond at low rates, below B', and go easy.

Interlocking interval-ratio schedules. Figure 3C shows the nature of interlocking interval-ratio schedules. The cumulative-record depiction shows that various possibilities exist, high rates paying off sooner but with more effort and low rates paying off with less effort but later. The schedule allows a whole range of compromises between working and waiting. The feedback function shown on the right (solid curve) is the sum of the ratio and interval feedback functions (dashed curve and lower line). It illustrates that the situation is a smooth blend of ratio- and interval-like payoffs. At relatively lower rates, the curvature indicates that the schedule tends toward what may be called the *corrective* tendency of interval
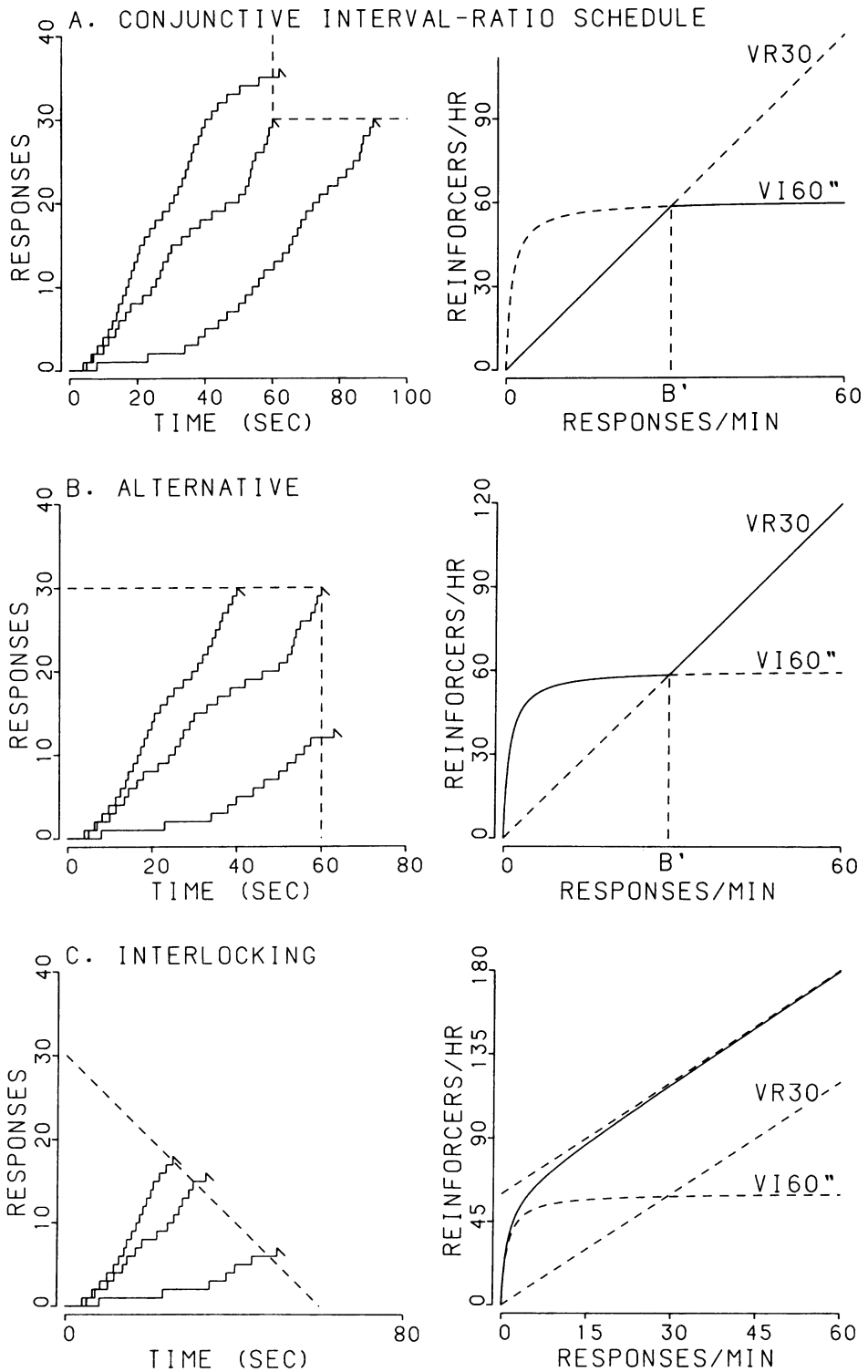
Figure 3.   Compound schedules: cumulative-record representations (left) and feedback functions (right).

A. ADJUSTING RATIO SCHEDULE

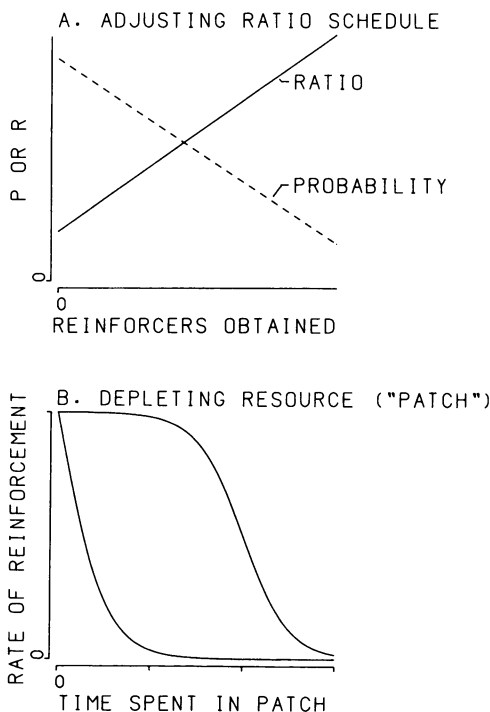## B. DEPLETING RESOURCE ("PATCH")

Figure 4. Adjusting ratio schedules as models of depleting resources or "patches." A: As more reinforcers are obtained, the ratio requirement increases or the probability that a response will be reinforced decreases. B: Depending on how the ratio or probability is adjusted, different patterns of depletion can result.

schedules—the lower the rate, the better the payoff. At relatively higher rates, the curve approaches the upper dashed line as an asymptote, which means that the situation shifts gradually toward ratio payoff. The feedback functions reveal that interlocking and alternative schedules resemble one another in offering choice between taking control and going easy.

Natural Resources

A different kind of complexity enters when we consider the situation organisms face in exploiting a resource in the natural environment. As one eats the berries in a berry patch, there are fewer and fewer to be found: the patch *depletes*. Depletion means that as the resource is consumed, it becomes progressively more difficult to obtain. For schedules, this means that as reinforcers are delivered,

the schedule requirement changes so as to make them less available.

Adjusting schedules as depleting resources. Figure 4 shows how adjusting schedules can model depleting resources. Since hunting, searching, and otherwise exploiting resources share with ratio schedules the property that rate of reinforcement depends directly on behavioral output, the schedule that models a natural resource is the adjusting ratio. Figure 4A shows adjustment of ratio schedules. Either the ratio requirement increases with number of reinforcers delivered or the probability of reinforcement decreases. Figure 4B shows the effect of the adjustments on the rate of reinforcement. With increasing time spent exploiting the resource, the rate of reinforcement falls, perhaps quickly and then slowly or perhaps slowly and then quickly, depending on the pattern of use. For example, systematic search for berries, working from a starting point and never retracing, will result in little or no decline in rate of finding berries until the patch has been completely covered, at which point rate of finding berries will decline precipitously. If a bird were to hunt for seeds in a patch of grass completely at random, then the rate of capturing seeds would decline rapidly at first and then ever more slowly as the number of seeds declined all over the patch (Baum, 1987).

The complexity of this type of situation arises because, unlike the schedules we considered before, adjusting schedules cannot be considered *stationary*; their parameters shift in time. This means that not only response rate but also time must be taken into account. For deriving feedback functions, at least two additional performance variables have to be considered: time spent responding ("time in") and time away from the patch. When one is searching for berries, interacting with the patch, that is time in; when one goes home or takes a nap under a nearby tree, no longer interacting with the patch, that is time away.

Interval schedules as depleting. Looked at this way, even an ordinary interval schedule shares some of the character of a depleting resource (Staddon, 1980).
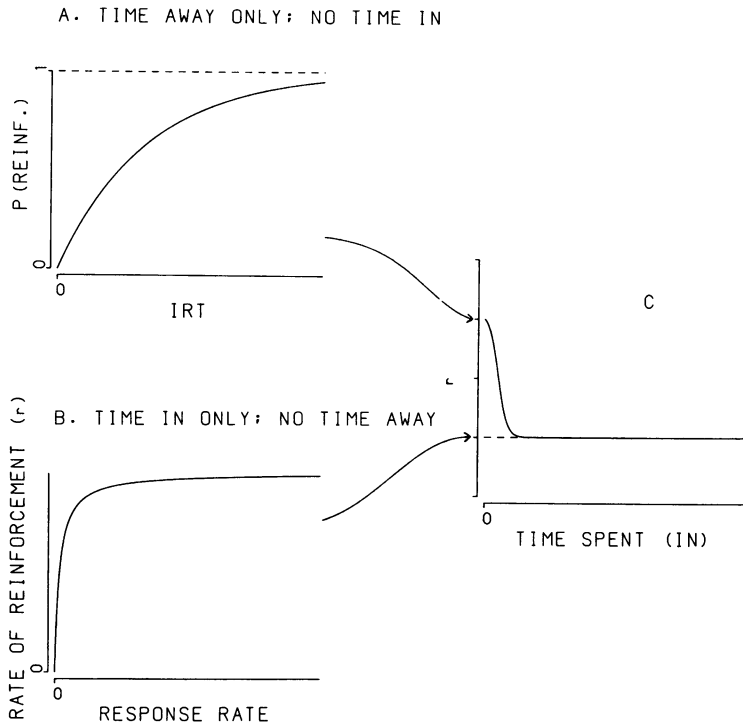
A. TIME AWAY ONLY; NO TIME IN

Figure 5.   Interval schedules as depleting. The most molecular view (A) includes only time away. The most molar view (B) includes only time in. Combining the two (C) reveals that interval schedules may share the property of depletion with natural resources.

Figure 5 illustrates this. Figure 5A shows differential reinforcement of IRTs in the usual molecular view of a variable-interval schedule. In such a view there is no time in, because responses are treated as if they have no duration, and the only time considered is the time between responses; so the IRT is considered time away. Figure 5B shows the feedback function of the molar view. Here, there is no time away, because rate of reinforcement and response rate are computed using all the available time. Hence, Figure 5B shows the effect of response rate during time in. Figure 5C shows how the two views combine when we consider both time away and time in. After some time away, there is a higher probability of reinforcement—hence a higher rate of reinforcement—at the beginning of time in. The height of this peak depends on the duration of time away (Figure 5A). As time in proceeds, the rate of reinforcement drops to a horizontal asymptote, the height of which depends on the re-

sponse rate during time in, as given in Figure 5B.

Feedback function for a patch. The feedback function for a depleting resource (patch) must take into account three performance variables: response rate while in the patch, time in the patch, and time away from the patch. This means the feedback function will be four-dimensional. Figure 6 shows only how overall rate of reinforcement would vary with time in, for a given time away and response rate. The curve indicates that there is an optimal duration of time in. The exact height and position of the maximum depend not only on time away, but on how the patch replenishes—whether quickly or slowly.

CONCLUSION

Molecular theories of behavior—those that rely solely on momentary events for explanation—have two great drawbacks. First, because present behavior arises
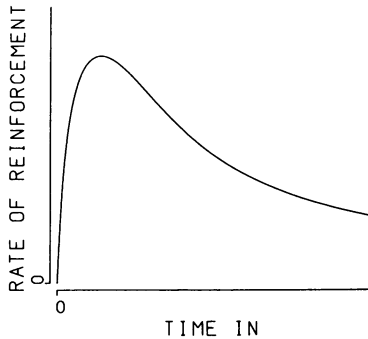
Figure 6.   Feedback function for a depleting patch: Effects of varying time spent responding ("time in").

from a history of events, extended through time, molecular theories require invention of hypothetical constructs like response strength and conditioned fear to represent the extended history in momentary events of the present. Since these hypothetical constructs are unmeasurable and vaguely defined, their explanatory power is more illusory than real. Second, molecular theories, even with the constructs, make only qualitative predictions. Were the properties of the constructs precisely defined, quantitative prediction might become possible, but the constructs would stand merely as surrogates of well-understood molar historical variables that can be specified in physical terms. Once defined, the hypothetical constructs become superfluous, because one can proceed to a direct molar analysis, relating molar behavioral variables to molar environmental variables.

With maturity, a science of behavior should be able to make quantitative predictions. Since quantitative predictions are possible only with molar laws, behavioral analysis can progress toward this goal only by looking beyond momentary events to molar variables and molar relations.

In the laboratory and in the field, a better understanding of contingencies can be had by thinking of their molar properties, by thinking of the impact of contingencies on history over a period of time. Instead of asking, "Does this contingency change behavior?" one can ask, "If we make rate (or amount) of reinforcement depend on response rate ac-

cording to this relation, how much does behavior change?" I have emphasized here the tactically prior problem of describing the environment in molar terms, relying on the concept of feedback functions (E-rules; Figure 1). The specification of functional relations (O-rules; Figure 1) will follow or accompany development of such molar descriptions. The result will be a science of behavior that is both powerful and practical.

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